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Elastic Constants of Bismuth

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The six adiabatic elastic stiffness constants of bismuth have been determined at 301°K by an ultrasonic pulse echo technique. The results are: $c_{11}=63.5$, $c_{33}=38.1$, $c_{44}=11.30$, $c_{66}=19.4$, $c_{14}=+7.23$, and $c_{13}=24.5$, all in units of 10^{10} d/cm². These values were redundantly determined by the measurement of 14 different velocities in four different single crystals of zone-purified bismuth. The velocities are believed accurate to better than 1%, the principal error arising from the uncertainty of the transducer transit time correction. The moduli are in poor agreement with the previously determined static elastic compliance constants reported by Bridgman. Some data on the velocity of sound in bismuth at 98° and at 4.2°K are also presented.

INTRODUCTION

THE acoustic determination of the adiabatic elastic constants of bismuth reported here was instigated in conjunction with measurements of the magneto acoustic-resistance of bismuth.[1] These initial observations were not in agreement with static values reported by Bridgman,[2] and, indeed, suggested that the latter were in error. However, the initial values were not redundant, nor even unambiguously determined. Because knowledge of the elastic constants is helpful in the theoretical investigation of the electronic band structure, the study was extended to provide more definitive results.

The primitive cell of bismuth is a rhombohedron ($\alpha=57°\,41'$) containing two atoms. The body diagonal of the rhombohedron has threefold symmetry and this trigonal axis is commonly designated as the z axis of the crystal. The plane perpendicular to the trigonal axis, containing the center of inversion, contains also three twofold axes and three bisectrices. To specify the other axes, we use the convention described by Cady,[3] according to which, a positive y axis is chosen to be along the projection of one edge of the primitive cell on the plane perpendicular to the [111] direction, and the positive x axis is then chosen along the binary axis which completes a right-handed orthogonal system. Such a detailed specification of axes is required in order to determine the sign of c_{14} unambiguously.

The six Voigt elastic constants for this class of crystal ($3m$) may be represented schematically by the matrix,

$$c_{ij}=\begin{vmatrix} c_{11} & c_{12} & c_{13} & c_{14} & 0 & 0 \\ c_{12} & c_{11} & c_{13} & -c_{14} & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ c_{14} & -c_{14} & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & c_{14} \\ 0 & 0 & 0 & 0 & c_{14} & \dfrac{c_{11}-c_{12}}{2} \end{vmatrix}. \quad (1)$$

We note in passing that according to the Laval-Raman[4] formalism, as modified by Joel and Wooster,[5] a more extended representation is required owing to their use of an unsymmetrical stress tensor. Previous tests[6–9] of this theory have been confined to piezoelectric materials in which the issue is confused by complications and experimental difficulties arising from electromechanical interactions. The symmetry in bismuth is such that a direct test of the Laval-Raman theory may be carried out in a simple manner, the test being limited only by the accuracy of the velocity of sound measurements used to determine the elastic constants.

EXPERIMENTAL PROCEDURE

The velocity of sound in the variously oriented crystals was determined by the pulse echo technique at 12 Mc using an apparatus previously described by Lazarus.[10] The delay line of a Dumond 256D oscilloscope was used to measure the difference in arrival time of successive echoes. The delay line was calibrated frequently during the course of the measurements by use of 10-μsec markers.

The principal source of error in this type of measurement arises from uncertainty about the correction time to be applied for the effective transit time in the transducer. This correction varies in magnitude depending on the relative velocities and lengths of the crystal and transducer. The acoustic mismatch at the crystal-transducer interface, which in turn is also a function of the type and thickness of adhesive used, also produces[11] a progressive distortion in the pulse shape of successive echoes. The distortion depends on the phase of the sound wave at the time of its incidence on the interface and thus depends on the frequency and length of the crystal for a given orientation. McSkimin[12] has de-

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[1] D. H. Reneker, Phys. Rev. 115, 303 (1959).

[2] P. W. Bridgman, Proc. Acad. Arts and Sci. 60, 305 (1925).

[3] W. G. Cady, *Piezoelectricity* (McGraw-Hill Book Company, Inc., New York, 1946), p. 23.

[4] J. Laval, Compt. rend. 242, 2502 (1956); C. V. Raman and K. S. Viswanathan, Proc. Ind. Acad. Sci. 42, 1 (1955); 42, 51 (1955).

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[9] H. Jaffe, Bull. Am. Phys. Soc. 4, 427 (1959).

[10] D. Lazarus, Phys. Rev. 76, 545 (1949).

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[12] H. J. McSkimin, IRE Trans. on Ultrasonics Eng. PGUE 5, 25 (1957).

veloped an ingenious resonant scheme for avoiding this inherent difficulty of the pulse echo technique. In our case, the errors were reduced to a minimum by the empirical procedure of determining the time interval between echoes with and without a dummy transducer attached to the reflecting end of the sample. The change in time interval produced by addition of the dummy was used to estimate the effective transit time correction for the identical driving transducer. The transit time correction so determined varied between 0.02 and 0.06 μsec. Most of the samples employed in these measurements were somewhat over 1 cm in length. Hence, the correction at most is of the order of 1% in velocity.

The bismuth single crystals were cut from a zone refined bar whose impurity concentration is estimated to be about 1 ppm. The crystal blocks approximately $1\times\frac{3}{4}$ in. in cross section were ground so their ends were flat and parallel to within 0.0001 in.

Most of the measurements of sound velocity were made at room temperature, actually 301°K. No effort was made to control the temperature accurately because of the small temperature coefficients of the elastic constants. Measurements made at helium and liquid nitrogen temperatures were made using the cryostat previously described by Reneker.[1]

The orientation of the crystals was determined to within $\pm1°$ by the standard Laue back reflection x-ray technique. The problem of ascertaining the directions of the positive x and positive y axes in the crystal was resolved as follows. One notes that on a stereographic projection, such as that given by Vickers[13] for the larger rhombohedral unit cell containing eight atoms per unit cell, the three positive x axes point in the $[01\bar{1}]$, $[\bar{1}01]$, and $[1\bar{1}0]$ directions and the three positive y axes point in the $[\bar{2}11]$, $[1\bar{2}1]$, and $[11\bar{2}]$ directions. For crystals not oriented along principal axes, we specify the orientation by polar angles θ and φ, where θ is the angle between the direction of propagation and the z axis and φ is the angle between the xz plane and the plane containing the z axes and the propagation direction. In our case, we are concerned only with $\theta=45°$, and $\varphi=\pm90°$. The differentiation between $=+90°$ and $=-90°$ is based on the fact that for $\varphi=-90°$ a very strong reflection corresponding to the (100) planes in Vicker's diagram occurs 11.5° from the center of the Laue picture. No such strong reflection occurs for $\varphi=+90°$. In addition, the identification may be checked by the occurrence of a relatively strong spot on the $\varphi=+90°$ picture corresponding to the $(11\bar{1})$ planes at an angular distance of about 26.5° from the center.

RESULTS

Fourteen independent velocities were measured at room temperature on an X-cut, a Y-cut, a $\theta=45°$, $\varphi=+90°$, and a $\theta=45°$, $\varphi=-90°$ crystal. The velocities are given in Table I. Using the method described

[13] W. Vickers, J. Metals 9, 827 (1957).

TABLE I. Observed velocities of sound on bismuth at 301°K.

Symbol	Direction of propagation	Velocity in 10^5 cm/sec	Mode
v_1	x axis	2.540±0.022	Longitudinal
v_2	x axis	1.550±0.009	Fast shear
v_3	x axis	0.850±0.004	Slow shear
v_4	y axis	2.571±0.018	Longitudinal
v_5	y axis	1.407±0.009	Shear polarized along x
v_6	y axis	1.022±0.006	Shear polarized along y
v_7	z axis	1.972±0.015	Longitudinal
v_8	z axis	1.074±0.011	Degenerate shear[a]
v_9	$\theta=45°$, $=+90°$	2.082±0.019	Longitudinal
v_{10}	$\theta=45°$, $=+90°$	1.522±0.017	Shear polarized along x
v_{11}	$\theta=45°$, $=+90°$	1.150±0.006	Shear polarized along $\theta=95°$
v_{12}	$\theta=45°$, $=-90°$	2.441±0.041	Longitudinal
v_{13}	$\theta=45°$, $=-90°$	0.910±0.003	Shear polarized along x
v_{14}	$\theta=45°$, $=-90°$	1.055±0.006	Shear polarized along 135°

[a] Owing to the degeneracy of the shear modes of propagation along the trigonal axis, internal conical refraction occurs and the transmitting crystal must be laterally displaced with respect to the receiving crystal in order to detect the pulses.

by Mason,[14] we may relate these velocities to the elastic constants by the equations[15–17]:

$$\rho v_1^2=c_{11} \tag{2}$$

$$\rho v_5^2=c_{66}=\tfrac{1}{2}(c_{11}-c_{12}) \tag{3}$$

$$\rho v_8^2=c_{44} \tag{4}$$

$$\rho v_7^2=c_{33} \tag{5}$$

$$\rho v_{13}^2=\tfrac{1}{2}(c_{66}+c_{44})-c_{14} \tag{6}$$

$$\rho v_{10}^2=\tfrac{1}{2}(c_{66}+c_{44})+c_{14} \tag{7}$$

$$\rho v_2^2=\tfrac{1}{2}[(c_{66}+c_{44})+\{(c_{44}-c_{66})^2+4c_{14}^2\}^{\frac{1}{2}}] \tag{8}$$

$$\rho v_3^2=\tfrac{1}{2}[(c_{66}+c_{44})-\{(c_{44}-c_{66})^2+4c_{14}^2\}^{\frac{1}{2}}] \tag{9}$$

$$\rho v_4^2=\tfrac{1}{2}[(c_{11}+c_{44})+\{(c_{44}-c_{11})^2+4c_{14}^2\}^{\frac{1}{2}}] \tag{10}$$

$$\rho v_6^2=\tfrac{1}{2}[(c_{11}+c_{44})-\{(c_{44}-c_{11})^2+4c_{14}^2\}^{\frac{1}{2}}]. \tag{11}$$

These are the equations which we have used to determine all the constants, except c_{13}. In addition, we have four additional relations

$$2\rho v_{12}^2=\tfrac{1}{2}(c_{11}+c_{33})+c_{44}+c_{14}+\{(\tfrac{1}{2}c_{11}-\tfrac{1}{2}c_{33}+c_{14})^2+(c_{13}+c_{44}+c_{14})^2\}^{\frac{1}{2}} \tag{12}$$

$$2\rho v_{14}^2=\tfrac{1}{2}(c_{11}+c_{33})+c_{44}+c_{14}-\{(\tfrac{1}{2}c_{11}-\tfrac{1}{2}c_{33}+c_{14})^2+(c_{13}+c_{44}+c_{14})^2\}^{\frac{1}{2}} \tag{13}$$

[14] W. P. Mason, *Physical Acoustics and the Properties of Solids* (D. van Nostrand Company, Inc., Princeton, New Jersey, 1958), p. 368.

[15] These formulas have also been given by Bhimasenacker but are repeated here because there are some typographical errors in his paper. Some formulas derived in the manner described by Mayer and Hiedemann are slightly different, apparently because they neglect certain nonzero off diagonal matrix elements of Eq. (1) in deriving their Eq. (7).

[16] J. Bhimasenacker, Proc. Ind. Acad. Sci. A29, 200 (1949).

[17] W. G. Mayer and E. A. Hiedemann, Acta Cryst. 12, 1 (1959).

TABLE II. Comparison of adiabatic elastic stiffness and compliance constants with isothermal values measured by Bridgman. The values in parentheses are calculated adiabatic values.

	Elastic constants of Bi	
	Echo method d/cm²	Bridgman d/cm²
C_{11}	63.5×10^{10}	62.9×10^{10} (63.3)
C_{12}	24.7×10^{10}	35.0×10^{10} (35.56)
C_{13}	24.5×10^{10}	21.1×10^{10} (21.6)
C_{33}	38.1×10^{10}	44.0×10^{10} (44.35)
C_{44}	11.30×10^{10}	10.84×10^{10}
C_{14}	$+7.23 \times 10^{10}$	-4.23×10^{10}
C_{66}	19.40×10^{10}	13.87×10^{10}
S_{11}	27.8×10^{-13}	26.9×10^{-13}
S_{12}	-10.2×10^{-13}	-14.0×10^{-13}
S_{13}	-11.3×10^{-13}	-6.2×10^{-13}
S_{33}	40.8×10^{-13}	28.7×10^{-13}
S_{44}	130.7×10^{-13}	104.8×10^{-13}
S_{14}	-24.4×10^{-13}	16.0×10^{-13}
S_{66}	76.0×10^{-13}	81.2×10^{-13}

$$2\rho v_9^2 = \tfrac{1}{2}(c_{11}+c_{33})+c_{44}-c_{14}+\{(\tfrac{1}{2}c_{11}-\tfrac{1}{2}c_{33}-c_{14})^2 + (c_{13}+c_{44}-c_{14})^2\}^{\frac{1}{2}} \quad (14)$$

$$2\rho v_{11}^2 = \tfrac{1}{2}(\tfrac{1}{2}c_{11}+\tfrac{1}{2}c_{33}+c_{44}-c_{14})-\{(\tfrac{1}{2}c_{11}-\tfrac{1}{2}c_{33}-c_{14})^2 + (c_{13}+c_{44}-c_{14})^2\}^{\frac{1}{2}}, \quad (15)$$

which have been used to determine c_{13}. The error in the latter is larger than for the other five constants since the error in constants derived solely from crystals not oriented along principal axes is inherently larger, being proportional to the error in misorientation rather than to the square as for a crystal oriented along principal axes.

If one proceeds to insert the measured velocities in Eq. (2) to Eq. (15) to determine the elastic constants, one should have eight redundant checks because there are only six unknowns and 14 equations. In particular, the trace of the Christofel determinant for the X-cut crystal (i.e., $v_1^2+v_2^2+v_3^2$) should equal that for the Y-cut crystal $v_4^2+v_5^2+v_6^2$. The two observed traces are 9.580×10^{10} cm²/sec² and 9.654×10^{10} cm²/sec², respectively. Similarly, the traces of $\varphi=+90°$ and $\varphi=-90°$ crystals should be equal. In this case, the observed values are 7.974×10^{10} cm²/sec² and 7.899×10^{10} cm²/sec², respectively. Further checks of similar nature are easily made by considering sums and differences of pairs of equations such as Eq. (6) and Eq. (7).

Our actual procedure was somewhat different. The values of c_{11}, c_{33}, c_{66}, c_{44}, and c_{14} were calculated from from Eq. (2)–Eq. (11). The value of c_{11} was then slightly readjusted within the experimental error to improve the over-all agreement with all the equations. No effort was made to use a least-square procedure in view of the labor involved. The values of these constants so determined are given in Table II. The value of c_{13} is determined by solution of Eq. (12) – Eq. (15). One obtains two solutions for each pair of the four equations. The two pairs of equations have only one common root, however, and only this root will yield a positive volume compressibility. This value is also given in Table II.

It should be noted that the sign of c_{14} is positive because of the convention adopted in defining our axes. For problems involving only the propagation of sound, this convention is of no importance. However, when one desires to investigate the interaction of these sound waves with electrons, it is imperative that the sign convention adopted for designating the elastic properties be the same as that for describing the Fermi surface of the carriers. We have adopted Cady's convention because of its widespread use in the description of quartz crystals.

The values of the elastic constants reported in Table II satisfy all the stability requirements on the lattice. These conditions are easily derived by requiring the determinant of the elastic constants, corresponding to the matrix of Eq. (1), and all its principal minors, be positive in the manner described by Alers and Neighbors.[18] In calculating the elastic constants, the density of bismuth was taken to be 9.80 g/cm³ in accordance with the latest precision determination of the lattice parameters by Barrett.[19]

In addition to the measurements at room temperature, all the elastic constants except c_{13} were determined also at the boiling point of liquid N_2 and of liquid He. These values are given in Table III. Corrections for the change in length of the bismuth with temperature were made with the thermal expansion data of Erfling.[20] The determination of c_{13} as a function of temperature was not carried out owing to the high uncertainty.

DISCUSSION

In Table II, for purposes of comparison with our values, we have recorded the static values of the elastic

TABLE III. Temperature dependence of the elastic constants in units of 10^{10} d/cm².

	300°K	80°K	4.2°K
c_{11}	63.5	68.6	68.7
c_{33}	38.1	40.6	40.6
c_{44}	11.3	12.7	12.9
c_{66}	19.4	22.4	22.5
c_{14}	7.23	8.05	8.44
c_{13}	24.5	···	···

TABLE IV. Comparison of adiabatic linear and volume compressibilities.

	Echo technique cm²/d	Bridgman[a] cm²/d
K_z	18.16×10^{-13}	16.13×10^{-13}
K_t	6.28	6.59
K_v	30.7	29.31

[a] Calculated from adiabatic values.

[18] G. P. Alers and J. R. Neighbors, J. Appl. Phys. 28, 1514 (1957)
[19] C. S. Barrett, Australian J. Phys. (to be published).
[20] H. D. Erfling, Ann. Physik 34, 136 (1939).

compliance constants s_{ij} determined many years ago by Bridgman.[2] The s_{ij} determined by Bridgman are isothermal values; the c_{ij} values reported here are adiabatic. We have, therefore, converted Bridgman's values of s_{ij} to isothermal c_{ij} by inverting the matrix of Eq. (1), and thence to adiabatic c_{ij}, by use of thermodynamic formulas of the standard type.[21] These values are also given in Table II. Finally, we have inverted our c_{ij} matrix to obtain adiabatic s_{ij} which within experimental error are the same as the isothermal values. It is to be noted that our values of the c_{ij} obtained by inverting Bridgman's s_{ij} are materially different from those reported by Betts *et al.*[22]

We focus attention on the comparison of our adiabatic measured values of c_{ij} with the adiabatic values of the c_{ij} computed from Bridgman's static s_{ij}. Several large discrepancies, far larger than the combined estimated experimental errors, are apparent for c_{33} and c_{12}. The origin of these discrepancies is difficult to ascertain. However, we note that Bridgman's values are only partially redundant, that several of his samples were reported as polycrystalline, and that the purity of his samples was dubious. We suspect a gross orientational error[23] (perhaps concealed as an occluded grain or an unnoticed twin) in one or more of his crystals. The discrepancy in the sign of c_{14} is only apparent and arises presumably because of a difference between Bridgman's unspecified axis convention and that used here.

A more rewarding comparison (see Table IV) is afforded by the agreement between Bridgman's directly observed adiabatic volume compressibility and that calculated from our data: the former is 29.3×10^{-13} cm^2/d and the latter, 30.7×10^{-13} cm^2/d. The difference between these two values is well within the combined experimental error.

We turn now to the question of whether the Voigt theory provides an adequate description of the elastic properties of bismuth. In view of the numerous redundant checks, the answer must certainly be affirmative to within the accuracy of our experiments. However, in view of the existing uncertainty over the applicability of the Laval-Raman theory to various crystals, it might be worthwhile to set some quantitative limits on the permitted deviations from the Voigt framework, at least, as applied to bismuth.

Briefly, the difference between the Voigt and the Laval-Raman theory may be stated as follows. The Voigt theory considers the case of a static (or homogeneous) strain in which the strain tensor may be separated into symmetric and antisymmetric components. The latter correspond to rigid body rotations which invoke no stress. The stress tensor may then be proved symmetric. In a dynamic and, consequently, inhomogeneous strain, the rotational part of the displacement has a spatial variation and hence requires torques to sustain the changes in angular momentum. It is therefore impossible to demonstrate that the stress tensor is symmetric. In the most general case, one requires 45 constants to relate the stress to the strain, after imposing exactness conditions, rather than 21 constants as in the Voigt theory. Wooster has shown that this number is further reduced by compatibility requirements to 39 constants. This number of constants is further reduced by the requirements of crystal symmetry so that for the simplest crystal classes (e.g., isotropic materials), there is no difference between the two theories.

Particularizing to the case of Bi, one finds that eight constants are required to specify the relation between stress components X_{ij} and strain components x_{ij}. In fact,[24,25]

$$X_{ij}=\begin{vmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & 0 & 0 & 0 \\ & d_{11} & -d_{13} & -d_{14} & -d_{15} & 0 & 0 & 0 \\ & & d_{33} & 0 & 0 & 0 & 0 & 0 \\ & & & d_{44} & \frac{d_{44}+d_{55}}{2} & 0 & 0 & 0 \\ & & & & d_{55} & \frac{d_{44}+d_{55}}{2} & d_{15} & d_{15} \\ & & & & & d_{55} & d_{14} & d_{14} \\ & & & & & & d_{66} & d_{66} \\ & & & & & & & d_{66} \end{vmatrix} x_{ij}. \tag{16}$$

[21] See J. F. Nye, *Physical Properties of Crystals* (Oxford University Press, Oxford, England, 1957), p. 283.
[22] D. D. Betts, A. B. Bhatia, and G. K. Horton, Phys. Rev. 104, 43 (1956).
[23] The discrepancy between the value for V_{12} previously reported by one of us and that in Table II was owing to a "false" trigonal axis being mislabeled in a Laue pattern.
[24] Note that this matrix differs slightly from that given by Raman and Viswanathan owing to a misprint in their article. We have used their notation which differs from that of LeCorre.
[25] Y. LeCorre, Bull. Soc. franc. Mines. Crist. 70, 80 (1957).

The Christofel determinantal equation for the velocities of propagation along the x axis of bismuth is

$$\begin{vmatrix} d_{11}-\rho v^2 & 0 & 0 \\ 0 & d_{66}-\rho v^2 & d_{15} \\ 0 & d_{15} & d_{55}-\rho v^2 \end{vmatrix} = 0, \quad (17)$$

and that for propagation along the y axis of bismuth is

$$\begin{vmatrix} d_{66}-\rho v^2 & 0 & 0 \\ 0 & d_{11}-\rho v^2 & -d_{15} \\ 0 & -d_{15} & d_{55}-\rho v^2 \end{vmatrix} = 0. \quad (18)$$

These particular cases are quite simple and differ from the corresponding equations for the Voigt theory in that d_{55} replaces c_{44} and d_{15} replaces c_{14}.

From Eq. (17), we have

$$d_{55}+d_{66}=\rho v_2^2+\rho v_3^2 \quad (19)$$

and, from Eq. (18)

$$d_{11}+d_{55}=\rho v_4^2+\rho v_6^2. \quad (20)$$

Furthermore, $d_{11}=c_{11}$ and $d_{66}=c_{66}$.

The sum, $v_2^2+v_3^2-v_5^2$, is $1.145\pm0.034\times10^{10}$ cm^2/sec^2, which is to be compared with the directly observed v_8^2, $1.153\pm0.017\times10^{10}$ cm^2/sec^2. It is clear that d_{44} and d_{55} cannot differ by more than 4%. Equation (20) does not yield as small an estimate for the possible difference between d_{44} and d_{55} owing to the large error introduced by the uncertainty in c_{11}.

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Dislocations in Indented Magnesium Oxide Crystals

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Dislocation rosette patterns produced by spherical and pyramidal indentors on the cleaved surfaces of magnesium oxide crystals were studied in detail. The three-dimensional arrangement of dislocation loops as deduced from the two-dimensional etching patterns is discussed. Cracks formed on $\{110\}_{90}$ planes around pyramidal indentations are believed to be due to the interaction of dislocations on $\{110\}_{45}$ planes. The temperature dependence of hardness was found to be related to the widening of dislocation bands, rather than to the distance of travel of leading dislocations. Some observations were also made on the pinning of dislocations and recovery at elevated temperatures, and on the interaction of dislocations with grown-in subboundaries.

INTRODUCTION

THE indentation hardness test is probably the simplest method of measuring the strength of materials. However, it is also the least understood test in terms of stress and strain distribution. Some progress has been made in analyzing the stress and strain distributions of several types of indentations using the continuum theory of plasticity.[1] However, the results cannot be applied directly to crystalline solids having well-defined slip systems.

In the last decade, a few attempts have been made to study the deformation mechanism associated with indentation. Tolansky and Nickols[2] studied several matrials by means of multiple-beam interference microscopy. Churchman, Geach, and Winston[3] investigated materials with a diamond structure. Smakula and Klein[4] used a prismatic punching method to study glide in ionic crystals. Votava, Amelinckx, and Dekeyser[5] employed an interferometric method to study indentation figures on cleavage faces of mica and NaCl.

With the advancement of dislocation theory and techniques of revealing dislocations in crystals in the past few years, it was thought possible to attain a better understanding of the deformation caused by indentation of a material with a simple crystalline structure. In this investigation, dislocation etching technique[6,7] was used to study the dislocation structures associated with various types of indentations at various temperatures in magnesium oxide crystals. This type of study may

[1] R. Hill, *The Mathematical Theory of Plasticity* (Oxford University Press, New York, 1950).
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